기말고사 응용편미분방정식 2002년 2학기

이것은 시험이니 다른 사람과 문제에 대하여 논의하지 않도록 하시오.
시험 문제에 대하여 질문이 있으면 12월 15일 전에 질문하기 바람 (12월 15일 부터 출장).
총 8 문제.
제출마감 : 12월 20일, 금요일

1. Solve Problem 6.6 of the Text.

2. (2-1) Let $\Omega := \mathbb{R}^n_+$, $n = 2, 3$, the upper half space. Let $u \in C^2(\overline{\Omega})$ be harmonic function in Ω such that $\frac{\partial u}{\partial n} = 0$ on $\partial\Omega$. In addition, we suppose that $u(x) = O(|x|^{-1})$ as $|x| \to \infty$. Show that $u \equiv 0$ in Ω. Can you prove this only assuming $u \in C^2(\Omega) \cap C^1(\overline{\Omega})$?

   (2-2) Let Ω be a bounded domain with a smooth boundary and let $u \in C^2(\Omega) \cap C^1(\overline{\Omega})$ be a harmonic function. If $u = 0$ on $\partial\Omega$, show that $u \equiv 0$. If $\frac{\partial u}{\partial n} = 0$ on $\partial\Omega$, show that u is constant in Ω.

3. Assume that a waveguide has a shape of square of side length a in xy-plane and infinitely long in z-direction. Suppose that this waveguide is surrounded by a perfect conductor. Find the eigenfrequencies of a plane wave propagating in z-direction.

4. Let Ω_j, $j = 1, 2, 3$, be regions in $\mathbb{R}^3$ defined by $\Omega_1 = \{z < 0\}$, $\Omega_2 = \{0 < z < a\}$, and $\Omega_3 = \{z > a\}$. Suppose that the dielectric constants for Ω_j are given by ϵ_j, μ_j, $j = 1, 2, 3$, Ω_3 is a perfect conductor and Ω_2, Ω_3 are insulators, and there is no surface charge density on the interfaces. Discuss about the reflection and refraction. The incident electric filed takes the same form as in the text.

5. Solve Problem 5.1 of the Text.

6. Solve Problem 7.3 of the Text.

7. We show that a harmonic function u in a domain Ω satisfies the mean value property, i.e.,

$$u(x) = \frac{1}{\text{Area}(\partial B(x,r))} \int_{\partial B(x,r)} u(y) dS \tag{1}$$

   for all $x \in \Omega$ and $r > 0$ with $B(x, r) \subset \Omega$. Show that the converse is also true. (Hint. Write (1) in the way we did in class. Then differentiate with respect to r.)

8. Find the solution of

$$\begin{cases} u_{tt} - u_{xx} = 0, \quad x > 0, t > 0, \\ u(0,t) = 0, \quad t > 0, \\ u(x,0) = g(x), \quad x > 0, \\ u_t(x,0) = f(x), \quad x > 0. \end{cases}$$

(Hint. Use the odd reflection over the t-axis and use d'Alembert formula.)