중간고사 응용편미분방정식 2002년 2학기

고사일: 10월 24일, 목요일

1. Solve the following initial boundary value problem:

$$\begin{cases} \dfrac{\partial^2 u}{\partial t^2} = \dfrac{\partial^2 u}{\partial x^2}, \quad 0 < x < 1,\ t > 0, \\ u(0,t) = 0, \quad t \geq 0, \\ u(1,t) = 0, \quad t \geq 0, \\ u(x,0) = 0, \quad 0 < x < 1, \\ \dfrac{\partial u}{\partial t}(x,0) = 2\sin(\pi x) - 3\sin(2\pi x), \quad 0 < x < 1. \end{cases}$$

2. Consider the following initial value problem:

$$\begin{cases} \dfrac{\partial^2 u}{\partial t^2} = 2\dfrac{\partial^2 u}{\partial x^2}, \quad -\infty < x < \infty,\ t > 0, \\ u(x,0) = f(x), \quad -\infty < x < \infty, \\ \dfrac{\partial u}{\partial t}(x,0) = 0, \quad -\infty < x < \infty. \end{cases}$$

   When f is supported in the intervals $[0,1]$ and $[2,3]$, draw the region in (x,t) half plane where the solution u vanishes.

3. Suppose that uniform traffic with density ρ_1 cars per kilometer approaches the end of a line of traffic stopped at a red light. Ahead of the red light there are no cars, while the stopped traffic is at its maximum density ρ_* cars per kilometer. At time $t = 0$, the red light turns green and the front of the line of stopped traffic begins to move forward. Our model for the resulting traffic density is

$$\rho_t + v_1(1 - \frac{2\rho}{\rho_*})\rho_x = 0, \quad -\infty < x < \infty,\ t > 0,$$

$$\rho(x,0) = \begin{cases} \rho_1 & \text{if } x \leq -L, \\ \rho_* & \text{if } -L < x < 0, \\ 0 & \text{if } x \geq 0. \end{cases}$$

   Assume that $\rho_1 = \frac{\rho_*}{2}$.

   (1) Find the characteristics for this initial value problem and sketch them.

   (2) Find the solution of the problem.