중간고사 I 　　　　 해석개론, 2000년 2학기

시험일 : 9월 28일

1. Let the function f_n be monotone increasing and continuous on $[0,1]$. Suppose that $F(x) = \sum_{n=1}^{\infty} f_n(x)$ converges for each $x \in [0,1]$. Show that F is continuous on $[0,1]$.

2. Suppose that a continuous function $f : [0,1] \to \mathbb{R}$ satisfies

$$\int_0^1 f(x)x^n dx = 0, \qquad n = 0, 1, 2, \cdots .$$

   Show that $f \equiv 0$.

3. Suppose that $\sum_{n=1}^{\infty} a_n = 0$. Show that

$$\lim_{x \to 1^-} \sum_{n=1}^{\infty} a_n x^n = 0.$$

4. Let $f(x) = L(x) + g(x)$ where $L : \mathbb{R}^n \to \mathbb{R}^m$ is a linear map and $g : \mathbb{R}^n \to \mathbb{R}^m$ satisfies $\|g(x)\| \le M\|x\|^2$ for all x and for some constant M. Show that $Df(0) = L$.

5. (5-1) Let $f : \mathbb{R}^n \to \mathbb{R}$ be differentiable at x_0. Show that the partial derivatives $\frac{\partial f}{\partial x_j}(x_0)$ exist.

   (5-2) Give an example showing that the converse is not true. Explain why your example is a counter-example.

6. Let $h(x,y) = f(x, u(x,y), v(x,y))$ where f, u, v are differentiable functions. Write down $Dh(x,y)$ in terms of f, u, v and their derivatives.

7. (7-1) Find the critical points of the function $f(x,y,z) = x^2 + y^2 + z^2 - xyz$ $(x > 0, y > 0, z > 0)$.

   (7-2) Determine the nature of the critical points found in (7-1).

8. Let $f : \mathbb{R} \to \mathbb{R}$ be differentiable. Assume there is no point $x \in \mathbb{R}$ such that $f(x) = f'(x) = 0$. Show that $S = \{x \in [0,1] : f(x) = 0\}$ is finite.