2000년 2학기 복소해석학, 대학원 강현배

강의내용

1. Riesz-Fredholm Theory of Compact Operators
2. Harmonic Functions
3. Basic Properties of Holomorphic Functions
4. Conformal Mapping
5. Zeros of Holomorphic Functions

참고문헌

[1] L.C.Evans, Partial Differential Equations, AMS, 1998

[2] W.Rudin, Real and Complex Analysis, 3rd Ed., McGraw-Hill, 1987

[3] L. Alfors, Complex Analysis, McGraw-Hill, 1979

[4] L. Hörmander, An introduction to complex analysis in several variables, North Holland, 1973

[5] J. Noguchi, Introduction to Complex Analysis, AMS, 1998

숙제 제출일: 3월 17일

1. Let $\{x_n\}$ a sequence in a Hilbert space X converging weakly to x. If K is a compact operator on X, show that Kx_n converges strongly to Kx.

2. Let Ω be a bounded open set in $\mathbb{R}^n$. Define opearator T_α on $L^2(\Omega)$ by

$$T_\alpha f(x) := \int_\Omega \frac{1}{|x-y|^\alpha} f(y)dy \qquad (x \in \Omega).$$

If $\alpha < n$, show that T_α is compact on $L^2(\Omega)$.

3. Let Ω be a bounded open set in $\mathbb{R}^n$. Let $X = H_0^1(\Omega)$ and $Y = L^2(\Omega)$. Suppose that a bounded linear operator $A : X \to Y$ satisfies the following: For each $\epsilon > 0$ there exists $C_\epsilon > 0$ such that

$$\|Au\|_Y \leq \epsilon\|u\|_X + C_\epsilon\|u\|_Y, \qquad \text{for all } u \in X.$$

Show that $A : X \to Y$ is compact.

숙제 제출일: 9월 28일

4. Let K be a compact self adjoint operator on a Hilbert space X. Show that there is an eigenvalue λ of A such that $|\lambda| = \|A\|$.

5. Let $\ell^2 := \{\{x_n\}| \ \sum_{n=1}^{\infty} |x_n|^2 < +\infty \ \}$ and $K : \ell^2 \to \ell^2$ be defined by

$$K(\{x_n\}) := \{\alpha_n x_n\}.$$

   Find a necessary and sufficient condition for K to be compact.

6. Use the spectral theorem to prove that any compact self adjoint operator is a (strong) limit of finite rank operators.

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숙제 제출일: 10월 9일

7. 함수 u가 $B = B_R(0)$에서 harmonic이고, radially symmetric, 즉 모든 unitary 변환 U에 대하여 $u(Ux) = u(x)$ $(x \in B)$를 만족하면, u는 상수함수임을 증명하시오.
8. Rudin, Real and Complex Analysis, 3rd Ed, chap 11, 9
9. If u satisfies the submean value property in Ω and C^2, show that $\Delta u \geq 0$ in Ω.

숙제 제출일: 10월 23일

10-13. Rudin, Real and Complex Analysis, 3rd Ed, chap 11, 13, 15, 16, 19

14. (1) If u is harmonic in Ω and $\overline{B(x,R)} \subset \Omega$, show that

$$|\nabla u(x)| \leq \frac{C}{R^n} \int_{\partial B(x,R)} |u(y)| d\sigma(y)$$

for some constant C independent of R and u.

(2) Prove the the Liouville theorem for harmonic functions: If u is harmonic and bounded on $\mathbf{R}^n$, then u is constant.

숙제 V 복소해석학(대학원), 2000년 2학기

숙제 제출일: 11월 9일

15-16. Rudin, Real and Complex Analysis, 3rd Ed, chap 10, (2), (5)

14. Compute the following integral:

$$\Phi(z) = \int_{-\pi}^{\pi} \frac{t}{z+e^{it}} dt \quad (|z| \neq 1).$$

15. Let f be an entire function with period 2π, i.e., $f(z+2\pi) = f(z)$ for all $z \in \mathbb{C}$.

    (1) Show that $f(z) = \displaystyle\sum_{n=-\infty}^{\infty} c_n e^{inz}$ where $c_n = \displaystyle\frac{1}{2\pi}\int_{-\pi}^{\pi} f(z)e^{-inz} dz$.

    (2) If there exist positive constants C and α such that

    $$|f(z)| \leq Ce^{\alpha|z|} \quad \text{for all } z,$$

    show that

    $$f(z) = \sum_{|n|\leq\alpha} c_n e^{inz}.$$

    (Hint. Change contours.)

16. Let $f \in H(\overline{U})$. Prove the following.

    (1) $f(z)\overline{f(\overline{z})}$ is holomorphic in U.

    (2) $\displaystyle\int_{-1}^{1} |f(x)|^2 dx \leq \frac{1}{2}\int_0^{2\pi} |f(e^{i\theta})|^2 d\theta,$

    (3) $0 \leq \displaystyle\sum_{j,k\geq 0} \frac{a_j \bar{a}_k}{j+k+1} \leq \pi \sum_{k\geq 0} |a_k|^2, \quad a_j \in \mathbb{C}.$

17. Suppose that $f \in H(\{\frac{1}{3} < |z| < 3\})$. Is it possible for f to satisfy the following relation?

$$f((1+\frac{1}{k})^2) = 1 + \frac{1}{k} \quad (k = 1, 2, \cdots)$$

중간고사 복소해석학(대학원), 2000년 2학기

2000년 10월 27일

1. Let $X = \{u \in C([0,1]) : \int_0^1 u(x)dx = 0\}$. Define for $u \in X$

$$Ku(x) := \int_0^x u(y)dy.$$

   (1-1) Show that X is a Banach space.

   (1-2) Show that K is a compact operator on X.

   (1-3) Show that for each $f \in X$ there exists unique $u \in X$ such that

$$u - Ku = f.$$

2. Let K be a compact self adjoint operator on a Hilbert space X. If the eigenspace corresponding to 0 is finite dimensional, show that there is an infinitely many eigenvalues of K.

3. Let Ω be a domain in $\mathbb{R}^n$ and let

$$X := \{u \in L^2(\Omega) | u : \text{ harmonic in } \Omega\}$$

   Show that X is a closed subspace of $L^2(\Omega)$.

4. Let

$$M_r P[f](x) := \sup_{y>0} |P[f](x,y)| \qquad \text{the radial maximal function}$$

$$Mf(x) := \sup_{r>0} \frac{1}{|B_r(x)|} \int_{B_r(x)} |f(y)| dy$$

the Hardy Littlewood maximal function.

   Show that

$$M_r P[f](x) \leq CMf(x).$$