

숙제 제출일: 10월 17일, 목요일

1. 교과서 문제 3.2
2. Find a shock wave solution for the following initial value problem, and then animate the result:

$$\begin{aligned}\rho_t + \rho^2 \rho_x &= 0, & -\infty < x < \infty, & t > 0, \\ \rho(x, 0) &= \begin{cases} 2 & \text{if } x \leq 0, \\ 1 & \text{if } x > 0. \end{cases}\end{aligned}$$

3. Find a rarefaction wave solution for the following initial value problem, and then animate the result:

$$\begin{aligned}\rho_t + \rho^2 \rho_x &= 0, & -\infty < x < \infty, & t > 0, \\ \rho(x, 0) &= \begin{cases} 1 & \text{if } x \leq 0, \\ 2 & \text{if } x > 0. \end{cases}\end{aligned}$$

숙제 제출일: 10월 24일, 목요일

1. 교과서 문제 7.1
2. Suppose that uniform traffic with density ρ_1 cars per kilometer approaches the end of a line of traffic stopped at a red light. Ahead of the red light there are no cars, while the stopped traffic is at its maximum density ρ_* cars per kilometer. At time $t = 0$, the red light turns green and the front of the line of stopped traffic begins to move forward. Our model for the resulting traffic density is

$$\rho_t + v_1 \left(1 - \frac{2\rho}{\rho_*}\right) \rho_x = 0, \quad -\infty < x < \infty, \quad t > 0,$$
$$\rho(x, 0) = \begin{cases} \rho_1 & \text{if } x \leq -L, \\ \rho_* & \text{if } -L < x < 0, \\ 0 & \text{if } x \geq 0. \end{cases}$$

Assume that $\rho_1 = \frac{\rho_*}{2}$.

- (1) Find the characteristics for this initial value problem and sketch them.
- (2) Find the solution of the problem.
- (3) Give brief discussion on your solution.