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1. Solve the following Dirichlet problem:

$$\begin{cases} \Delta u = 0 \quad \text{in } \Omega = \{(x,y) : 0 < x < 1, 0 < y < 1\}, \\ u(x,0) = \sin \pi x,\ u(x,1) = \sin 2\pi x,\ u(0,y) = u(1,y) = 0, \quad 0 \le x \le 1,\ 0 \le y \le 1. \end{cases}$$

2. Show that for any $f \in C_0^\infty(\mathbb{R}^2)$

$$\frac{1}{8\pi} \int_{\mathbb{R}^2} |x-y|^2 \log|x-y| \Delta^2 f(y) dy = f(x).$$

3. Let Ω be a bounded domain with a smooth boundary. If $u \in C^2(\Omega) \cap C(\overline{\Omega})$ satisfies $\Delta u = 0$ in Ω and $\frac{\partial u}{\partial n} = 0$ on $\partial\Omega$, show that $u = \textit{constant}$ on Ω.

4. Let φ be a continuous function in $\mathbb{R}^n$ satisfying

$$|\varphi(x)| \le \frac{C}{(|x|+1)^{n+1}}.$$

Define $\varphi_\epsilon(x) := \frac{1}{\epsilon^n}\varphi(\frac{x}{\epsilon})$ for $\epsilon > 0$. If $\int_{\mathbb{R}^n} \varphi(x) dx = A$, show that for any bounded continuous function f

$$f * \varphi_\epsilon(x) \to f(x) \quad \text{as } \epsilon \to 0.$$