해석개론I-004, 중간고사 2 2008.5.14

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1. Write down definitions of the following notions:

   (1-1) Compact sets

   (1-2) Connected sets

   (1-3) Uniform continuity

2. Let A be a compact subset of a metric space M and $\{U_i\}$ be an open cover of A. Show that there is an $r > 0$ such that for each $y \in A$, $D(y, r) \subset U_i$ for some i.

3. Let $f : \mathbb{R} \to \mathbb{R}$ be a bounded function. Prove that f is continuous if and only if the graph of f is a closed subset of $\mathbb{R}^2$.

4. Let

$$f(x) = \begin{cases} x^2 \sin \dfrac{1}{x}, & x \neq 0, \\ 0, & x = 0. \end{cases}$$

   (4-1) Show that f is differentiable at 0.

   (4-2) Is f' continuous at 0?

5. Let f be an increasing function of the interval $[a, b]$.

   (5-1) Show that f is integrable on $[a, b]$.

   (5-2) Show that the discontinuity of f is at most countable.

   (5-3) Construct an increasing function on $[a, b]$ which is discontinuous at countably many points.