Real Analysis, Spring Semester, 2009

HomeWork

HomeWork 1 (due: April 29, 2009)

1. P.96, 24

2. P.104, 10

3. P.111, 16

4. Let f be a L^p function on $[0, 1]$.

(1-1) If $1 \leq p \leq \infty$, show that for every ϵ there is a step function φ such that

$$\|f - \varphi\|_p < \epsilon.$$

(1-2) If $1 \leq p < \infty$, show that for every ϵ there is a continuous function ψ such that

$$\|f - \psi\|_p < \epsilon.$$

(1-3) If $p = \infty$, show that (1-2) is not true.

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HomeWork 2 (due: May 11, 2009) (수업시간에 제출. 늦은 숙제는 받지 않음)

P. 123, 8. P. 127, 17. P. 134, 21. Rudin P. 93, 13.

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HomeWork 3 (due: June 1, 2009)

1. If f is an L^1-function on $\mathbb{R}^1$, prove that

$$\lim_{y \to 0} \|f - f_y\|_1 = 0,$$

where f_y is the translation of f by y, *i.e.*, $f_y(x) = f(x - y)$.

2. Show that there is no function $h \in L^1(T)$ such that $f \circ h = f$ for all $f \in L^1(T)$. (Hint: Riemann-Lebesgue Lemma).

3. For $f \in L^1(T)$ and for $n = 1, 2, 3, \ldots$, define

$$f^n = f \circ \cdots \circ f \quad (n \text{ times}).$$

Prove that

$$\lim_{n\to\infty} \|f^n\|_1^{1/n} = \max_n |\hat{f}(n)|.$$

4. Prove that

$$\sum_{n=1}^{\infty} \frac{1}{n^4} = \frac{\pi^4}{90}.$$

5. Let f be a continuous function on $[0,1]$ satisfying $f(0) = f(1) = 0$. Solve the following problem:

$$\begin{aligned}
&\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2} + f, \\
&u(0,t) = u(1,t), \quad t > 0, \\
&u(x,0) = \frac{\partial u}{\partial t}(x,0) = 0.
\end{aligned}$$