Recent Advances in Nonlocal Kinetic, Fluid and Diffusive PDEs

Utop Ubless Hotel Jeju, South Korea
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Organized by

José A. Carrillo (Imperial College London, UK)
Young-Pil Choi (Inha University, South Korea)

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National Research Foundation of Korea
Basic Research Lab
Institute of Applied Mathematics, Inha University

Program

Recent Advances in Nonlocal Kinetic, Fluid and Diffusive PDEs
Date: August 19 – 23, 2019
Venue: Utop Ubless Jeju, South Korea

August 19 (Monday)

09:00 – 09:20	Registration
09:20 – 09:30	Opening Hyeonbae Kang (Inha University)

Chair: Young-Pil Choi

09:30 – 10:10	Seung-Yeal Ha Emergent behaviors of Cucker-Smale flocks on a Riemannian manifold
10:10 – 10:50	Xiongtao Zhang Complete predictability of the Cucker-Smale model on the real line with singular interactions
10:50 – 11:10	Coffee Break

Chair: Changhui Tan

11:10 – 11:50	In-Jee Jeong Singular vortex patches
11:50 – 12:30	Jinhuan Wang Critical mass for a two-species chemotaxis model with two chemicals in R^2

12:30 – 14:00 Lunch

Chair: Seung-Yeal Ha

14:00 – 14:40 Razvan Fetecau
Self-organization on Riemannian manifolds

14:40 – 15:20 Giacomo Albi
Kinetic optimal control problems of collective behavior

15:20 – 15:40 Coffee Break

Chair: Piotr Gwiazda

15:40 – 16:20 Oliver Tse
Dynamical-variational transport costs: Towards a framework for generalized gradient flows

16:20 – 17:00 Kyungkeun Kang
A regularity condition and temporal decays for chemotaxis-fluid equations

August 20 (Tuesday)

09:00 – 09:30 Registration

Chair: Giacomo Albi

09:30 – 10:10 Shi Jin
Random batch methods for interacting particle systems

10:10 – 10:50 Li Wang
Variational numerical methods for Wasserstein gradient flow

10:50 – 11:10 Coffee Break

Chair: Zhaoyin Xiang

11:10 – 11:50 Moon-Jin Kang
Uniform stability of shock layers for 1D hyperbolic-parabolic systems

11:50 – 12:30 Agnieszka Swierczewska-Gwiazda
Measure-valued – strong uniqueness property to various system of conservation laws

12:30 – 14:00 Lunch

Chair: Moon-Jin Kang

14:00 – 14:40 Li Chen
Global boundedness and hair trigger effect of solutions for a nonlocal reaction-diffusion equation in population dynamics

14:40 – 15:20 Zhaoyin Xiang
On a chemotaxis-Navier-Stokes system with mixed boundary conditions

15:20 – 15:40 Coffee Break

Chair: Tong Yang

15:40 – 16:20 Jan Haskovec
Cucker-Smale type models with delay

16:20 – 17:00 Jaemin Park
Symmetry in steady and stationary solutions of active scalar equations

18:00 – 20:00 Banquet

August 21 (Wednesday)

Chair: Renjun Duan

09:30 – 10:10 Donghyun Lee
On the collisional kinetic model with boundary conditions

10:10 – 10:50 Satoshi Taguchi
Inversion of the transverse force on a spinning sphere moving in a rarefied gas

10:50 – 11:10 Coffee Break

Chair: Donghyun Lee

11:10 – 11:50 Tong Yang
Some mathematical theories on the Boltzmann equation without angular cutoff

11:50 – 12:30 Seok-Bae Yun
Regularity estimates for the gain term of the spatially homogeneous Boltzmann equation for relativistic particles

12:30 – 14:00 Lunch

14:00 - Free Discussion

August 22 (Thursday)

09:00 – 09:30 Registration

Chair: Seok-Bae Yun

09:30 – 10:10	Lingbing He On semi-classical limit of spatially homogeneous quantum Boltzmann equation
10:10 – 10:50	Renjun Duan The Boltzmann equation with large-amplitude initial data in bounded domains
10:50 – 11:10	Coffee Break

Chair: Li Wang

11:10 – 11:50	Kazuo Aoki A kinetic model for a polyatomic gas with temperature-dependent specific heats and its application to shock-wave structure
11:50 – 12:30	Stephane Brull Local discrete velocity grids for multi-species rarefied flow simulations
12:30 – 14:00	Lunch

Chair: Kyungkeun Kang

14:00 – 14:40	Jihoon Lee Asymptotic behavior of the solutions to the coral fertilization model
14:40 – 15:20	Changhui Tan Eulerian dynamics with alignment interactions
15:20 – 15:40	Coffee Break

Chair: Peter A. Markowich

15:40 – 16:20 Piotr Gwiazda
A two-species hyperbolic-parabolic model of tissue growth

16:20 – 17:00 Yong Jung Kim
Diffusion model for heterogeneous environment

August 23 (Friday)

Chair: Jose A. Carrillo

09:30 – 10:10 Peter A. Markowich
On PDE models for transportation networks

10:10 – 10:50 Qi Wang
Steady states and bump solutions of Keller-Segel chemotaxis models with degenerate diffusion

10:50 – 11:10 Coffee Break

Chair: Yong Jung Kim

11:10 – 11:50 Hideki Murakawa
Marco- and microscopic models for cell-cell adhesion

11:50 – 12:30 Jan Peszek
A gradient flow leading to Cucker-Smale-type alignment dynamics

12:30 – 14:00 Lunch

14:00 - Free Discussion

Kinetic optimal control problems of collective behavior

Giacomo Albi

August 19 – 23, 2019

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Abstract

Kinetic models arising in the mathematical description of collective behavior have been studied in a wide spectrum of applications such as animal behaviour, cellular aggregation, opinion dynamics and human crowd motion. Our research addresses the design of external control actions which are able to steer a large system of interacting agents towards prescribed stable patterns. We address this challenge by means of optimal control techniques, thus minimizing an energy measure of both the control and the state of the system. In order to circumvent the solution of a large scale non-linear optimization problem arising at the microscopic level we use a multiscale framework by introducing a mean-field optimal control problem, where the agent dynamics evolves according to a nonlinear, nonlocal kinetic equation.

In order to solve numerically the mean-field optimal control problem we propose two stochastic approaches based on the efficient simulation of a Boltzmann-like model. The first approach solves the optimal control at the level of binary interactions, generating a sub-optimal mean-field feedback laws. The second approach tackles directly the control of the Boltzmann-like model producing a consistent approximation of the mean-field optimal control.

In both cases we will show that the associated stochastic algorithms are extremely efficient to cope with the solution of the mean-field control problem, even for moderate dimension. Different numerical examples will be presented in the context of consensus dynamics, and swarming models.

A kinetic model for a polyatomic gas with temperature-dependent specific heats and its application to shock-wave structure

Kazuo Aoki[1], Shingo Kosuge[2], and Hung-Wen Kuo[1]

August 19 – 23, 2019

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Abstract

The ellipsoidal statistical (ES) model of the Boltzmann equation for a polyatomic gas with constant specific heats (calorically perfect gas), proposed by Andries *et al.* [P. Andries *et al.*, Eur. J. Mech. B/Fluids **19**, 813 (2000)], is extended to a polyatomic gas with temperature-dependent specific heats (thermally perfect gas). Then, the new model equation is used to investigate the structure of a plane shock wave with special interest in CO_2 gas, which is known to have a very large bulk viscosity, and in the case of relatively strong shock waves. A numerical analysis, as well as an asymptotic analyses for large bulk viscosity, are performed in parallel to our previous paper [S. Kosuge and K. Aoki, Phys. Rev. Fluids **3**, 023401 (2018)], where the structure of a shock wave in CO_2 gas was investigated using the ES model for a polyatomic gas with constant specific heats. From the numerical and analytical results, the effect of temperature-dependent specific heats on the structure of a shock wave is clarified.

Local discrete velocity grids for multi-species rarefied flow simulations

Stéphane Brull

August 19 – 23, 2019

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Abstract

The aim of thie method is to develop a deterministic numerical method for kinetic equations that is adaptative w.r.t the velocity variable. In the classical methods, the velocity grids are chosen identical for each space point and constant in time. Moreover, the construction of such a global grid is based only on the initial conditions. However, in the context of rarefied gas flows, such as the airflow around the walls of a shuttle, important gradients of velocity and temperature can appear.

The idea of this work is to define dynamic sets of discrete velocities independantly for every species and every space discretization point. These sets are then defined according to the local value of the partial moments of each distribution function, by assuming them to be Maxwellian distributions. To adapt dynamically to the gradients of macroscopic quantities, partial moments are computed by the use of conservation laws obtained by taking the moments of the discrete kinetic equations. This formulation allows an implicit treatment of the relaxation operator leading to an Asymptotic-Preserving scheme for the Euler regime. The method is then implemented and tested on the BGK model for gas mixtures that has been proposed by Andries, Aoki and Perthame.

Global boundedness and hair trigger effect of solutions for a nonlocal reaction-diffusion equation in population dynamics

Li Chen

August 19 – 23, 2019

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Abstract

We investigate the global boundedness and the hair trigger effect of solutions for the nonlinear nonlocal reaction-diffusion equation

$$\partial_t u = \Delta u + u^\alpha(1 - J * u^\beta), \quad \text{in } \mathbb{R}^N \times (0,\infty),\ N \geq 1 \tag{1}$$

with $\alpha, \beta \geq 1$ and $u(x,0) = u_0(x)$. Under some assumptions on J, we prove that for any nonnegative and bounded u_0, if $1 \leq \alpha < \alpha^*$ or $1 \leq \alpha \leq \frac{\beta+1}{2}$ with

$$\alpha^* = \begin{cases} 1+\beta, & N = 1,2, \\ 1+\frac{2\beta}{N}, & N \geq 3, \end{cases} \tag{2}$$

the problem has a global bounded classical solution. Furthermore, we show that for $1 \leq \alpha < \alpha^*$, Lunder the assumption that $\int_{B(x,\delta)} u_0^{1-\alpha}(s)ds \in L^\infty(\mathbb{R}^N)$ for some $\delta > 0$ and $\|u_0\|_{L^\infty(\mathbb{R}^N)} \leq 1$, the solution converges to 1 uniformly on any compact subset of $\mathbb{R}^N$, i.e. $u \equiv 0$ is not stable, which is known as the hair trigger effect. A formal deduction of the model from a mesoscopic formulation is provided as well. We also perform numerical simulations of the above nonlocal reaction-diffusion equation and investigate the effect of convolution kernels and initial data on the solution behavior. These motivate a discussion about some conjectures arising from this model and further issues to be studied in this context. This is a joint work with Jing Li and Christina Surulescu.

The Boltzmann equation with large-amplitude initial data in bounded domains

Renjun Duan

August 19 – 23, 2019

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Abstract

The talk is devoted to constructing the global solutions around global Maxwellians to the initial-boundary value problem on the Boltzmann equation in general bounded domains with isothermal diffuse reflection boundaries. We allow a class of non-negative initial data which have arbitrary large amplitude and even contain vacuum. The result shows that the oscillation of solutions away from global Maxwellians becomes small after some positive time provided that they are initially close to each other in L^2. This yields the disappearance of any initial vacuum and the exponential convergence of large-amplitude solutions to equilibrium in large time. The isothermal diffuse reflection boundary condition plays a vital role in the analysis. The most key ingredients in our strategy of the proof include: (i) $L^2_{x,v}$–$L^\infty_x L^1_v$–$L^\infty_{x,v}$ estimates along a bootstrap argument; (ii) Pointwise estimates on the upper bound of the gain term by the product of L^∞ norm and L^2 norm; (iii) An iterative procedure on the nonlinear term. This is a joint work with Yong Wang (Chinese Academy of Sciences).

Self-organization on Riemannian manifolds

Razvan Fetecau

August 19 – 23, 2019

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Abstract

We consider an aggregation model that consists of an active transport equation for the macroscopic population density, where the velocity has a nonlocal functional dependence on the density, modelled via an interaction potential. We set up the model on general Riemannian manifolds and provide a framework for constructing interaction potentials which lead to equilibria that are constant on their supports. We consider such potentials for two specific cases (the two-dimensional sphere and the two-dimensional hyperbolic space) and investigate analytically and numerically the long-time behaviour and equilibrium solutions of the aggregation model on these manifolds. Equilibria obtained numerically with other interaction potentials and an application of the model to aggregation on the rotation group $SO(3)$ are also presented.

A two-species hyperbolic-parabolic model of tissue growth

Piotr Gwiazda

August 19 – 23, 2019

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Abstract

Models of tissue growth are now well established, in particular in relation to their applications to cancer. They describe the dynamics of cells subject to motion resulting from a pressure gradient generated by the death and birth of cells, itself controlled primarily by pressure through contact inhibition. In the compressible regime we consider, when pressure results from the cell densities and when two different populations of cells are considered, a specific difficulty arises from the hyperbolic character of the equation for each cell density, and to the parabolic aspect of the equation for the total cell density. For that reason, few a priori estimates are available and discontinuities may occur. Therefore the existence of solutions is a difficult problem.

In a common work with Benoît Perthame [1] and Agnieszka Świerczewska-Gwiazda we established the existence of weak solutions to the model with two cell populations which react similarly to the pressure in terms of their motion but undergo different growth/death rates.

References

[1] P. Gwiazda, B. Perthame, A. Świerczewska-Gwiazda *A two species hyperbolic-parabolic model of tissue growth*, to appear in Comm. Partial Differential Equations (2019)

Emergent behaviors of Cucker-Smale flocks on a Riemannian manifold

Seung Yeal Ha

August 19 – 23, 2019

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Abstract

In this talk, we will introduce a new Cucker-Smale(CS) model on smooth Riemannian manifolds using the concepts of covariant derivative and parallel transport, and we also study its emergent dynamics under an a priori assumption on the energy functional. For Euclidean space, our proposed model coincides with the original Cucker-Smale model. As concrete examples, we consider three Riemannian manifolds with constant mean curvatures: the unit sphere, the unit circle and the Poincaré half-plane, and provide explicit reductions from the proposed general model for those spaces via explicit formulars for the covariant derivative and parallel transport. This talk is based on the joint work with Doheon Kim (KIAS) and Franz Wilhelm Schloder (University of Milano-Bicocca)

Cucker-Smale type models with delay

Jan Haskovec

August 19 – 23, 2019

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Abstract

We study the impact of communication or reaction delays on the long-time asymptotics of Cucker-Smale type systems. For a model where agents interact with each other through normalized communication weights, we provide sufficient conditions for asymptotic flocking, i.e., convergence to a common velocity vector, based on a construction of a suitable Lyapunov functional. Moreover, a rigorous limit passage to the mean-field limit of the particle system as the number of particles tends to infinity will be presented. For a model without normalization, we present new stability estimates for the particle flow, relating suitable delayed and non-delayed quantities. Numerical simulations of the discrete system with few particles will provide further insights into the flocking and oscillatory behaviors of the particle velocities depending on the size of the time delay. This is a joint work with Young-Pil Choi (Inha U.) and Ioannis Markou (IACM Forth).

On semi-classical limit of spatially homogeneous quantum Boltzmann equation

Lingbing He

August 19 – 23, 2019

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Abstract

The quantum Boltzmann equations for Fermi-Dirac and Bose-Einstein statistics proposed by Uehling and Uhlembeck (after Nordheim) should be derived from the evolution of real Fermions and Bosons in the so called weak-coupling limit. Since Fokker-Planck-Landau equation is the effective equation associated with a dense and weakly interacting gas of classical particles, it is not surprising that the semi-classical limits of the solutions to quantum Boltzmann equations are expected to be solutions to the Fokker-Planck-Landau equation. In this talk, we will show that these limits can be justified mathematically. It is based on a joint work with Xuguang Lu and Marrio Pulvirenti.

Singular vortex patches

I.-J. Jeong

August 19 – 23, 2019

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Abstract

Vortex patches are solutions to the 2D Euler equations that are given by the characteristic function of a bounded domain that evolves with time. If the boundary of the domain is smooth initially, it remains smooth for all time. On the other hand, we consider patches with corner singularities. We have shown that, depending on whether the initial patch satisfies an appropriate rotational symmetry condition or not, the corner structure may propagate for all time or lost immediately. In the rotationally symmetric case, as a consequence of the well-posedness theory, we are able to construct patches with interesting dynamical behavior as time goes to infinity, such as cusp and spiral formation. When the symmetry is absent, we present some ill-posedness results which suggest that a corner singularity generically cusps instantaneously for $t > 0$. This is a joint work with T. M. Elgindi (University of California San Diego).

Random Batch Methods for Interacting Particle Systems

Shi Jin

August 19 – 23, 2019

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Abstract

We develop random batch methods for interacting particle systems with large number of particles. These methods use small but random batches for particle interactions, thus the computational cost is reduced from $O(N^2)$ per time step to $O(N)$, for a system with N particles with binary interactions. On one hand, these methods are efficient Asymptotic-Preserving schemes for the underlying particle systems, allowing N-independent time steps and also capture, in the $N \to \infty$ limit, the solution of the mean field limit which are nonlinear Fokker-Planck equations; on the other hand, the stochastic processes generated by the algorithms can also be regarded as new models for the underlying problems. For one of the methods, we give a particle number independent error estimate under some special interactions. Then, we apply these methods to some representative problems in mathematics, physics, social and data sciences, including the Dyson Brownian motion from random matrix theory, Thomson's problem, distribution of wealth, opinion dynamics and clustering. Numerical results show that the methods can capture both the transient solutions and the global equilibrium in these problems.

This is a joint work with Lei Li (Shanghai Jiao Tong University) and Jian-Guo Liu (Duke University)

A regularity condition and temporal decays for chemotaxis-fluid equations

Kyungkeun Kang

August 19 – 23, 2019

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Abstract

We consider two dimensional chemotaxis equations coupled to the Navier–Stokes equations. We present a new localized regularity criterion that is localized in a neighborhood at each point. Secondly, we establish temporal decays of the regular solutions under the assumption that the initial mass of biological cell density is sufficiently small. This is a joint work with Myeongju Chae and Jihoon Lee.

Uniform stability of shock layers for 1D hyperbolic-parabolic systems

Moon-Jin Kang

August 19 – 23, 2019

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Abstract

In this talk, I will briefly present a new approach (introduced in a collaboration with Alexis Vasseur) to get the contraction estimate for any large perturbations of shock layers to the 1D barotropic Navier-Stokes system. Since the contraction estimate is uniformly in the strength of the viscosity coefficient, this uniformity implies the uniqueness result of entropy shocks to the isentropic Euler system in a class of inviscid limits from the Navier-Stokes system. Surprisingly, the new approach is quite robust, since it can be applied to more general 1D viscous hyperbolic systems of two conservation laws. For example, I will present a result on contraction estimate of viscous traveling waves to a 1D hyperbolic-parabolic system arising from a chemotaxis model in tumour angiogenesis.

Diffusion model for heterogeneous environment

Yong-Jung Kim

August 19 – 23, 2019

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Abstract

Thermal diffusion (or the Soret effect) is an example that shows Fick's law is not enough to explain diffusion phenomena in spatially heterogeneous environments. Indeed, there are long standing debates about the correct diffusion model in such cases. In this talk, diffusion models are introduced which are derived from random walk system, discrete kinetic system, stochastic processes, and others. We will see that, as long as the system is revertible, they ends up in the same diffusion equation. We will also consider its application to biological phenomena such as chemotaxis theory.

On the collisional kinetic model with boundary conditions

Donghyun Lee

August 19 – 23, 2019

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Abstract

In this talk, we discuss the Boltzmann equation, a mathematical model for rarefied gas theory. The Boltzmann equation has been studied for a long time, but many boundary condition problems are still open. Main issue of the problem is very singular behaviors generated by interactions between gas and boundary effects. To avoid this issue, low regularity setup will be introduced and we show global well-posedness and convergence to equilibrium asymptotically.

Asymptotic behavior of the solutions to the coral fertilization model

Jihoon Lee

August 19 – 23, 2019

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Abstract

In this talk, we consider a following coral fertilization model

$$\begin{cases} \partial_t e + (u \cdot \nabla) e - \Delta e = -\epsilon(se), \\ \partial_t s + (u \cdot \nabla) s - \Delta s = \chi \nabla \cdot (s \nabla \Delta^{-1} e) - \epsilon(se), \end{cases} \quad \text{in} \quad (x,t) \in \mathbb{R}^d \times (0, \infty), \tag{1}$$

where $e \geq 0$, $s \geq 0$, and u denote the density of egg gametes, sperm gametes and the divergence free sea velocity of sea fluid, respectively. When $d = 2, 3$, we show that there exists a global regular solution to this system. When $d = 2$, 3, we have the following temporal decay estimates

$$\|e(t)\|_{L^p(\mathbb{R}^d)} \leq \frac{C}{t^{\frac{d}{2}\left(1-\frac{1}{p}\right)}}, \quad p \in (1, \infty], \tag{2}$$

and

$$\|s(t)\|_{L^p(\mathbb{R}^d)} \leq \frac{C}{t^{\frac{d}{2}\left(1-\frac{1}{p}\right)}}, \quad p \in (1, \infty). \tag{3}$$

We also consider the system coupled with the incompressible Navier-Stokes equations. This talk is based on the joint work with Myeongju Chae and Kyungkeun Kang.

On PDE models for transportation networks

Peter A. Markowich

August 19 – 23, 2019

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Abstract

TBA

Macro- and microscopic models for cell-cell adhesion

Hideki Murakawa

August 19 – 23, 2019

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Abstract

Cell-cell adhesion and cell sorting processes are essential in organ formation during embryonic development and in maintaining multicellular structure. In order to deal with cell sorting of aggregations of large number of cells with complicated shapes, such as aggregations of neurons in brain, it is necessary to consider cell population dynamics models. Here, we propose a nonlocal advection-diffusion system as a possible mathematical model for such phenomena. Numerical experiments illustrate that the proposed model is able to replicate different types of phenomena observed in cell sorting experiments. Furthermore, this macroscopic model can be applied to elucidate real problems in life sciences. This work is based on a joint work with Jose A. Carrillo, Makoto Sato, Hideru Togashi and Olena Trush.

On the other hand, there are some cell sorting phenomena which should be captured by individual based models such as the mosaic cellular patters in sensory epitheliums. We also propose a microscopic model for understanding the pattern formations of such phenomena. This part is based on a joint work with Sayaka Katsunuma, Rhudaina Z. Mohammad, Karel Svadlenka and Hideru Togashi.

Symmetry in Steady and Stationary Solutions of Active Scalar Equations

Jaemin Park

August 19 – 23, 2019

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Abstract

We study whether all stationary solutions of 2D Euler equation must be radially symmetric, if the vorticity is compactly supported or has some decay at infinity. Our main results are the following:

(1) On the one hand, we are able to show that for any non-negative smooth stationary vorticity that is compactly supported (or has certain decay as $|x| \to \infty$), it must be radially symmetric up to a translation.

(2) On the other hand, if we allow vorticity to change sign, then by applying bifurcation arguments to sign-changing radial patches, we are able to show that there exists a compactly-supported, sign-changing smooth stationary vorticity that is non-radial.

We have also obtained some symmetry results for uniformly-rotating solutions for 2D Euler equation, as well as stationary/rotating solutions for the SQG equation. The symmetry results are mainly obtained by calculus of variations and elliptic equation techniques. This is a joint work with Javier Gomez-Serrano, Jia Shi and Yao Yao.

A gradient flow leading to Cucker-Smale-type alignment dynamics

Jan Peszek

August 19 – 23, 2019

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Abstract

It is well known that the 1D Cucker-Smale particle system can be transformed into a variant of Kuramoto system with constant ”natural frequencies” acting as parameters. Inspired by this transformation, we introduce a change of variables for the kinetic 1D Cucker-Smale model that leads to a parametrized continuity equation in $\mathbb{R}^2 = \mathbb{R}_x \times \mathbb{R}_\omega$ ($\mathbb{R}_x$ is the physical x-space and $\mathbb{R}_\omega$ is the ω-space of ”natural frequencies” acting as space of parameters).

The talk is dedicated to the analysis of such a parametrized continuity equation as a gradient flow in a carefully chosen topology of nonoptimal transport referred to as *fibered topology*. Fibered topology arises as an optimal transport problem with constrains forcing the transport to occur only along the ω-fibers of $\mathbb{R}^2$.

This procedure can be naturally generalized to the multidimensional case leading to a Cucker-Smale-type model of alignment dynamics that coincides with the Cucker-Smale model in 1D.

The talk presents a joint project with Javier Morales (University of Maryland, College Park) and David Poyato (University of Granada).

Measure-valued – strong uniqueness property to various systems of conservation laws

Agnieszka Świerczewska-Gwiazda

August 19 – 23, 2019

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Abstract

In the last years measure-valued solutions started to be considered as a relevant notion of solutions if they satisfy the so-called measure-valued – strong uniqueness principle. This means that they coincide with a strong solution emanating from the same initial data if this strong solution exists. This property has been examined for many systems of mathematical physics, including incompressible and compressible Euler system, compressible Navier-Stokes system et al. and there are also some results concerning general hyperbolic systems. I will discuss a unified framework for general systems, that would cover the most interesting cases of systems, as well as Euler-alignment and Euler-Poisson system.

Inversion of the transverse force on a spinning sphere moving in a rarefied gas

Satoshi Taguchi

August 19 – 23, 2019

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Abstract

The flow around a spinning sphere slowly moving in a rarefied gas is investigated on the basis of kinetic theory with a special interest in the force and torque acting on the sphere. More precisely, an asymptotic analysis of the Boltzmann system for small Mach numbers, incorporating inner and outer expansions, is carried out in the case where the ratio between the equatorial surface velocity and the translational velocity of the sphere, also known as the spin parameter, is finite, and the expressions for the force and torque are derived up to the second order of the Mach number. The expressions are valid for arbitrary Knudsen numbers as long as the Reynolds number is small. It is shown that (i) the transverse force on the sphere, also known as the lift, occurs as a result of the interplay between the flow past a sphere and the flow induced around the sphere caused by the spin, and that (ii) the spin does not affect the drag and torque up to the order considered. For given translational and angular velocities of a sphere, the lift is modulated by a function depending on the Knudsen number, that is, the ratio between the molecular mean free path and the sphere radius. A representation formula of this function in terms of the solutions of two elementary flow problems of a rarefied gas, namely a uniform flow past a sphere and a flow around a rotating sphere, is derived by making use of the duality property of the linearized Boltzmann equation (i.e. symmetry relation). Then the transverse force is numerically obtained on the basis of the BGK model of the Boltzmann equation under the diffuse reflection boundary condition on the sphere. The result shows a striking feature: for given translational and angular velocities of a sphere, the transverse force varies monotonically with the Knudsen number, and above a certain critical Knudsen number it exhibits a negative lift. The threshold of the negative lift, which is independent of the translational and angular velocities in the present weakly nonlinear theory, is determined in the case of the BGK model under the diffuse reflection condition. In the present analysis, a transition of the transverse force on a spinning sphere in term of the Knudsen number is clearly observed, which gives a new insight to what is known as the inverse Magnus effect previously reported for a free molecular gas by several authors.

This work was supported by JSPS KAKENHI Grant Numbers 25820041 and 17K06146.

Eulerian dynamics with alignment interactions

Changhui Tan

August 19 – 23, 2019

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Abstract

The Euler-Alignment system arises as a macroscopic representation of the Cucker-Smale model, which describes the flocking phenomenon in animal swarms. The nonlinear and nonlocal nature of the system bring challenges in studying global regularity and long time behaviors. In this talk, I will discuss the global wellposedness of the Euler-Alignment system with three types of nonlocal alignment interactions: bounded, strongly singular, and weakly singular interactions. Different choices of interactions will lead to different global behaviors. I will also discuss interesting connections to some fluid dynamics systems, including the fractional Burgers equation, and the aggregation equation.

Dynamical-variational transport costs: Towards a framework for generalised gradient flows

Oliver Tse

August 19 – 23, 2019

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Abstract

Evolution equations in spaces of measures describe a wide variety of natural phenomena. The theory for such evolutions has seen tremendous growth in the last decades, of which resulted in general metric space theories for analysing variational evolutions—evolutions driven by one or more energies/entropies. On the other hand, physics and large-deviation theory suggest the study of *generalised* gradient flows—gradient flows with non-homogeneous dissipation potentials—which are not covered in metric space theories.

In this talk, we introduce dynamical-variational transport costs (DVTs)—a large class of large-deviation inspired functionals that provide a variational generalisation of existing transport distances—to remedy this deficiency. The role in which these objects play in the theory of *generalised* gradient flows will be illustrated with an example on Markov jump processes. Finally, open questions and challenges will be mentioned.

Critical mass for a two-species chemotaxis model with two chemicals in $\mathbb{R}^2$

Jinhuan Wang

August 19 – 23, 2019

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Abstract

I this talk, we consider a two-species chemotaxis model with two chemicals in $\mathbb{R}^2$. Let m_1, m_2 be the initial mass of the two species respectively. The critical mass of the model is established as a curve of the form $m_1 m_2 - 4\pi(m_1 + m_2) = 0$. That is to say the solutions exist globally if $m_1 m_2 - 4\pi(m_1 + m_2) < 0$, and the finite time blow-up of solutions may occur if $m_1 m_2 - 4\pi(m_1 + m_2) > 0$.

Variational numerical methods for Wasserstein gradient flow

Li Wang

August 19 – 23, 2019

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Abstract

We develop variational methods for nonlinear equations with a gradient flow structure. Such equations arise in applications of a wide range, such as porous median flows, material science, animal swarms, and chemotaxis. Our method builds on the JKO framework, which evolves the equation as a gradient flow with respect to the Wasserstein metric. As a result, our method has built-in positivity preserving, entropy decreasing properties, and overcomes stability issue due to the strong nonlinearity and degeneracy. We further modify the variational formulation by adding a Fisher information regularization so that second order information can be used to accelerate the convergence.

Steady states and bump solutions of Keller-Segel chemotaxis models with degenerate diffusion

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August 19 – 23, 2019

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Abstract

Chemotaxis is the mechanism by which unicellular or multicellular organisms direct their movements in response to a stimulating chemical in the environment. Bacterial chemotaxis was discovered by T. W. Engelmann and W. Pfeffer in 1880s, and over one century's research has illustrated its importance in many physiological processes. In the 1970s, E. Keller and L. Segel proposed a system of two coupled partial differential equations to describe the traveling bands of *E. coli* in a capillary tube filled with oxygen. The intuitively simple Keller-Segel model possesses very rich spatial-temporal dynamics (e.g., finite-time blow-up, traveling wave, spike steady states) and has achieved great academic success over the past few decades.

An important extension of the classical Keller-Segel model is to include a density-dependent diffusion function, assuming that the random cell motility is anti-crowding and decreases as the population thins. In this talk, we consider a Keller-Segel model with porous medium diffusion and study the effects of chemotaxis on the existence and stability of nonconstant and particularly compactly-supported steady states. A novelty in the quadratic diffusion structure is utilized to obtain the explicit formulas of steady states in a one-dimensional interval. We also give a complete hierarchy of their energies on the bifurcation diagram, and with that being said, the half-bump has the least energy while the constant steady state has the largest energy among all steady states. In the large limit of chemotaxis rate, the cell population density converges to a single or several Dirac-delta functions, which can be adopted to model the cell aggregation phenomenon. We will present numerics to demonstrate the theoretical results and some interesting phenomena such as phase transition, focusing and defocusing process within this model.

On a chemotaxis-Navier-Stokes system with mixed boundary conditions

Zhaoyin Xiang

August 19 – 23, 2019

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Abstract

In this talk, we investigate the large time behavior of strong solutions to a chemotaxis-Navier-Stokes system in an unbounded domain with finite depth and mixed boundary conditions. Based on some uniform *a priori* estimates obtained by using the anisotropic L^p technique and the subtle elliptic estimates, we first establish the global existence of strong solution around the equilibrium state $(0, c_{\text{air}}, 0)$ with the help of the continuity arguments, where c_{air} is the saturation value of oxygen inside the fluid. Then we use De Giorgis technique and cutoff method to show that such a solution will converge to $(0, c_{\text{air}}, 0)$ with an explicit convergence rate in the chemotaxis-free case. Our assumptions and results are consistent with the experimental descriptions and the numerical analysis. This is a joint work with Yingping Peng.

Some mathematical theories on the Boltzmann equation without angular cutoff

Tong Yang

August 19 – 23, 2019

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Abstract

In this talk, we will present two recent works on the Boltzmann equation without angular cutoff. One is about the regularizing effect of the Debye-Yukawa potential for the spatially homogeneous Boltzmann equation, and another one about perturbative solution in the spatially inhomogeneous case with algebraic decay tail.

Regularity estimates for the gain term of the spatially homogeneous Boltzmann equation for relativistic particles.

Seok-Bae Yun

August 19 – 23, 2019

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Abstract

In this talk, we establish two types of regularity-gain type estimates for the gain term of the spatially homogeneous Boltzmann equation for relativistic particles. More precisely, we derive a gain-of-differentiability esetimate and a gain-of-boundedness estimate for the gain part of the collision operator. We then discuss applications of theses estimates: Rigorous proof of the H-theorem for relativistic particles, propagation of uniform, or exponential upper bounds, and asymptotic stabilization to relativistic equilibrium. This is from joint works with Jin-Woo Jang and Robert M. Strain.

Complete predictability of the Cucker-Smale model on the real line with singular interactions

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Abstract

In this talk, we consider Cucker-Smale mdoel (C-S) with singular communication on the real line. For long range interaction, we will prove the finite many collisions and thus derive the uniqueness of the solution. For short range interaction, we will show the avoidance of collisions by constructing the lower bound of the distance between particles. Therefore, the solution is analytic and unique. Moreover, we will also show the sufficient and necessary condition of multi-cluster formation in the singular case.